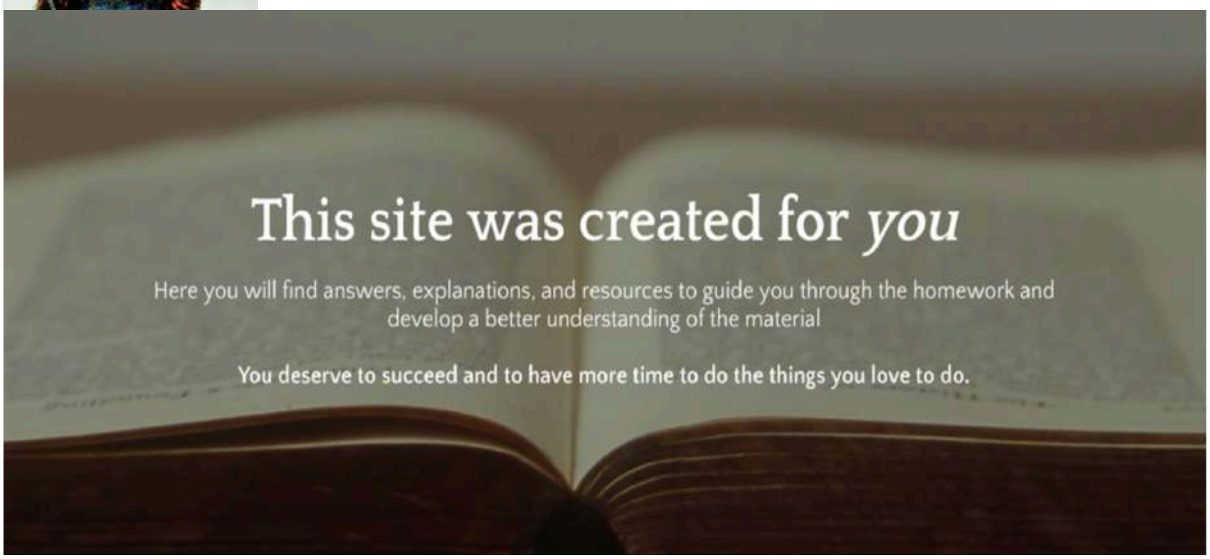
Highlander Help



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Good Luck!

Math 211: Calculus IIIA   Common Exam 1   March 1, 2017

Name: (print) [redacted]

Student ID number: [redacted]

Section Number: [redacted]

Signature*: [redacted]

*My signature affirms that this examination is completed in accordance with the NJIT Academic Integrity Code.

Instructions:

Please complete the problems on the following pages in the space provided. If you need additional space to work, please use the back of the previous page. All work must be shown in order to receive full credit. Answers without explanation will receive *no* credit. The use of books, notes, calculators, or any other external sources of information is not permitted during this examination.

pls check these ones

Question	Points	Score
1	6	3
2	4	4
3	15	10
4	10	10
5	15	8
6	15	12
7	15	0
8	10	10
9	10	10
Total:	100	~~67~~ +2

+2

~~67~~

69

$\text{proj}_v u \left(\frac{|u \cdot v|}{|v \cdot v|}\right)\vec{v}$

1. (6 points) Answer the following true/false questions. Showing work not required.

   (a) **T** **F** The (vector) projection of $\langle 3, 17, -19\rangle$ onto $\langle 1, 2, 2\rangle$ is equal to the (vector) projection of $\langle 3, 17, -19\rangle$ onto $\langle -5, -10, -15\rangle$.

   (b) **T** **F** The angle between the vectors $\langle 1, -3, 7\rangle$ and $\langle -4, 6, 1\rangle$ is obtuse (greater than $\frac{\pi}{2}$).

   (c) **T** **F** The tangent plane to $x^2 - y^2 + 4z^2 = 1$ at the point $(1, 2, 1)$ is $x - 2y + 4z = 1$.

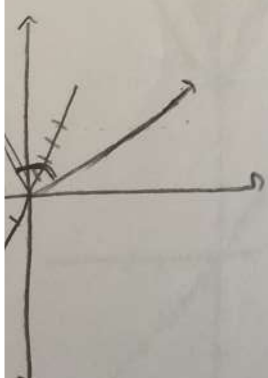
2. (4 points) Match the sketch of each quadric surface with the appropriate equation. You do not need to justify your choices.

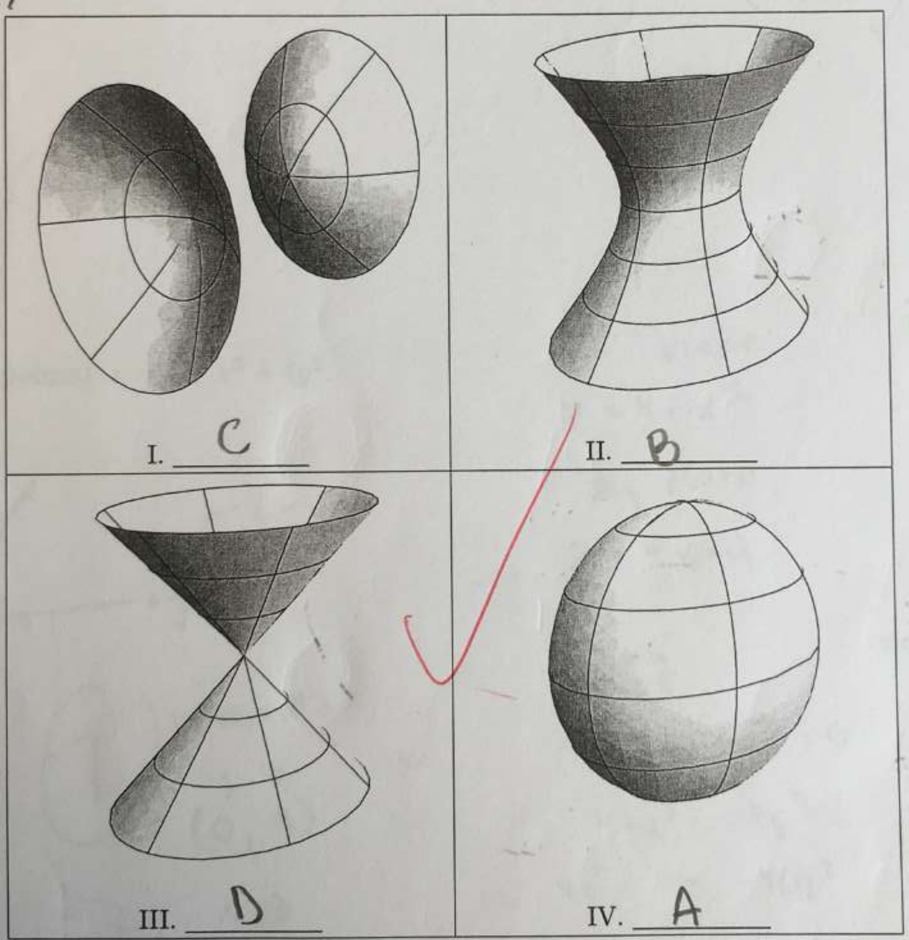
(a)	$x^2 + 2y^2 + z^2 = 1$	(b)	$x^2 + 2y^2 - z^2 = 1$
(c)	$x^2 - 2y^2 - z^2 = 1$	(d)	$x^2 + 2y^2 - z^2 = 0$

$-4 \quad -18 + 7$

$\left(\frac{1(-4)+6(-3)+7(1)}{\sqrt{59}\ \sqrt{53}}\right)$

$\left(\frac{-15}{\sqrt{59}\sqrt{53}}\right)$





$\frac{\pi}{2}$

90

$1(-4) + 6(-3) + 7(1)$

$-4 \quad -18 + 7$

$\cos\theta = -\frac{1}{4}$

$1 + 9 + 49$

$16 + 36 + 1$

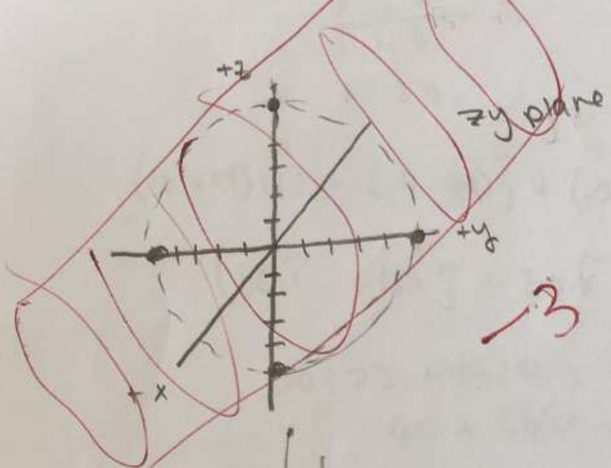
$\cos^{-1}\left(\frac{-15}{\sqrt{59}\ \sqrt{53}}\right)$

3. Sketch the solutions to the following equations, considered as surfaces in three-dimensional space. Include the intercepts of the surfaces with the three coordinate (x, y, z) axes, if any.

(a) (6 points) $y^2 + z^2 = 25$.

$y^2 = 25 - z^2$

$0^2 = 25 - 5^2$

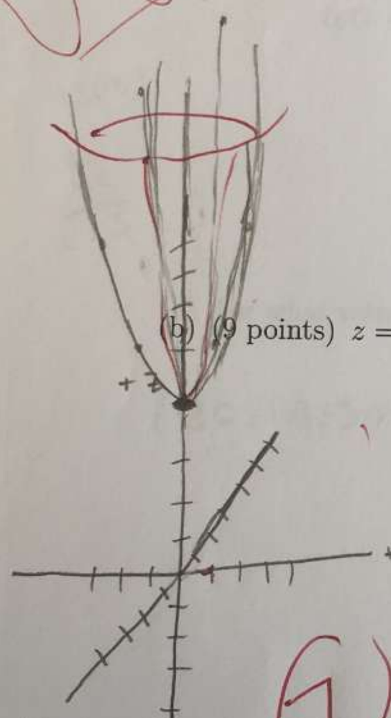


-3

z-intercepts at (y, z) (0, 5) (0, -5) ✓

y intercepts at (y, z) (5, 0) (-5, 0) ✓

3



(b) (9 points) $z = 4 + x^2 + 4y^2$.

xz plane

$z = 4 + x^2$

zy plane

$z = 4 + 4y^2$

z cannot be negative

-2

xy

no xy plane

$4 + x^2 + 4y^2 = 0$

np

$4 + x^2 = -4y^2$

$4 + \quad = -4(1)^2$

7

intercepts

y z (0, 4) ✓

+2

x z (0, 4)

$4 + x^2 + 4y^2 = 0$

$\neq$

4. (10 points) Consider the three vectors $\mathbf{p} = \langle 1, 4, 7\rangle$, $\mathbf{q} = \langle 7, -2, 4\rangle$, and $\mathbf{r} = \langle 2, 3, t\rangle$, where the value of t remains variable.

(a) For what value of t is $\mathbf{r}$ perpendicular to $\mathbf{p} \times \mathbf{q}$?

⊥ orthogonal

$$\begin{array}{ccc} i & j & k \\ \hline 1 & 4 & 7 \\ 7 & -2 & 4 \end{array}$$

$(16+14)(\hat{i}) - (4-49)\hat{j} + (-28-2)\hat{k}$

$(30\hat{i} + 45\hat{j} - 30\hat{k}) \cdot (2, 3, t) = 0$

$30(2) + 45(3) - 30t = 0$

$60 + 135 - 30t = 0$

$-30t = -195$

$t = \frac{195}{30}$

$45(3)$

$90 + 45 = 135$

$(-2-28)$

$\begin{array}{ccc} i & j & k \\ 1 & 4 & 7 \\ 7 & -2 & 4 \end{array}$

$(4-49)$

$4(4)-(14)$

$(16+14)$

$450 + 60 + 30 = 540$

$135 + 60 = 195$

$\begin{array}{ccc} i & j & 30k \\ 1 & 4 & 7 \\ 7 & -2 & 4 \end{array}$

$(16+14)i - (4-49)j \quad (-2-28)k$

$30 \quad +45 \quad -30$

(b) For what value of t is $\mathbf{r}$ parallel to $\mathbf{p} \times \mathbf{q}$? ×

$(30, 45, -30) \times (2, 3, t) = 0$

$$\begin{array}{ccc} i & j & k \\ 30 & 45 & -30 \\ 2 & 3 & t \end{array}$$

$(45t + 90)\,i - (30t + 60)\,j + (90 - 90)\hat{k} = 0$

$45t + 90 - 30t - 60 = 0$

$15t - 30 = 0$

$t = -2$

5. (15 points) (a) Decompose the vector $\mathbf{v} = \mathbf{j}$ into its projection onto $\mathbf{w} = \mathbf{i} + \mathbf{j} + \mathbf{k}$ and a remainder $\mathbf{r}$ that is perpendicular to the projection.

$$\text{proj}_v u = \left(\frac{u \cdot v}{v \cdot v}\right) \vec{v}$$

$$\left(\frac{(0,1,0)\cdot(1,1,1)}{(1,1,1)\cdot(1,1,1)}\right)(1,1,1)$$

$$\text{proj}_w v = \left(\frac{v \cdot w}{w \cdot w}\right) \vec{w}$$

$$\frac{1}{3}(1,1,1)$$

$$\frac{1}{3}i, \frac{1}{3}j, \frac{1}{3}k$$

vector remainder

$$\bar{r} = (0,1,0) - \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$$

$$-\frac{1}{3}, \frac{2}{3}, -\frac{1}{3}$$

$\sim = ?$

dot product → orthogonal

plane line

(b) Show that the line $x = 1 + t$, $y = -1 + 2t$, $z = 2 + 2t$ is parallel to the plane $2x - 2y + z = 1$ then find the distance between them.

$$\frac{|\vec{PS} \times \vec{v}|}{|\vec{v}|}$$

$x = 1 + t$

$y = -1 + 2t$

$z = 2 + 2t$

plane: $2x - 2y + z = 1$

$\frac{8}{15}$

n↑ $(2, -2, 1)$

$(1- \ )(-1- \ )(2-$

$(1 - 0)\ 1$

$(-1 - 0)\ -1$

$(2 - 1)\ 1$

n↑ $(1,2,2)$ parallelism

pt $(1,-1,2)$

dot product

$2) + 2(-2) + 1(2) = 0$

cross product

$$\begin{matrix} 1 & 2 & 2 \\ 2 & -2 & 1 \end{matrix}$$

$$\cancel{\frac{|(1,-1,1) \times (1,2,2)|}{\sqrt{9}}}$$

$(2+4)i - (1-4)j + (-2-4)k$

$$\begin{matrix} i & j & k \\ 1 & -1 & 1 \\ 1 & 2 & 2 \end{matrix}$$

$(-2-2)i - (2-1)j + (2+1)k$

$-4i - (1)j + 3k$

$$\boxed{\frac{\sqrt{26}}{3}}$$

$16 + 9 + 1$

$-4^2 + 1^2 + 3^2$

$-\sin t + \sin t + t\cos t$

6. (15 points) Consider the curve defined by a particle moving according to the equation $\mathbf{r}(t) = (\sin t - t\cos t)\mathbf{i} + (\cos t + t\sin t)\mathbf{j} + t^2\mathbf{k}$.

(a) Find the velocity vector, the acceleration vector *and* speed of the particle at the time $t = \frac{\pi}{2}$.

$v(t) = (t\sin t))i + (\;\; t\cos t)j + 2t\,k$

$a(t) = (\sin t + t\cos t)\hat{i} + (\cos t - t\sin t)\hat{j} + 2\hat{k}$

$v(t) @ \frac{\pi}{2} = t \qquad \frac{\pi}{2}\sin(\frac{\pi}{2}) + \cancel{2\sin(\frac{\pi}{2})} + \frac{\pi}{2}\cos\frac{\pi}{2} + 2(\frac{\pi}{2})$

$\boxed{\frac{\pi}{2} + \pi}$

(b) Find the tangent line to the particle path at $t = \frac{\pi}{4}$.

$t = \frac{\pi}{4} \qquad \frac{\pi}{4}\frac{\sqrt{2}}{2}\hat{i} + \frac{\pi}{4}\frac{\sqrt{2}}{2}\hat{j} + \frac{\pi}{2}k$

$$\boxed{\begin{aligned} x &= \frac{\sqrt{2}\pi}{8}t + \left(\frac{\sqrt{2}}{2} - \frac{\pi\sqrt{2}}{8}\right) \\ y &= \frac{\sqrt{2}\pi}{8}t + \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}\pi}{8}\right) \\ z &= \frac{\pi}{2}t + \left(\frac{\pi^2}{16}\right) \end{aligned}}$$

$r(t) = \sin\frac{\pi}{4} - \frac{\pi}{4}\cos\frac{\pi}{4}\hat{i} + \cos\frac{\pi}{4} + \frac{\pi}{4}\sin\frac{\pi}{4}\hat{j} + (\frac{\pi}{4})^2\hat{k}$

$(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}\pi}{8}) \qquad (\frac{\sqrt{2}}{2} + \frac{\sqrt{2}\pi}{8}) + (\frac{\pi^2}{16})$

5

(c) Between $t = 0$ and $t = T$, the particle moves a total distance of 3 units. What is T?

$$3 = \int_0^T \sqrt{(t\sin t)^2 + (t\cos t)^2 + (2t)^2}$$

$$\sqrt{t^2\sin t^2 + 4t^2 + t^2\cos t^2}$$

$$t\sqrt{1+4}$$

5

$$3 \int_0^T \sqrt{5}\,t$$

$$\left.\frac{\sqrt{5}\,t^2}{2}\right|_0^T = 3$$

$$\sqrt{\frac{3(2)}{\sqrt{5}}}$$

$$\boxed{\sqrt{\frac{6}{5^{\frac{1}{2}}}}}$$

7. (15 points) A ball is thrown northward into the air from the origin in xyz-space (the xy plane represents the ground, with the positive y-axis pointing north). The initial velocity of the ball is

$$\mathbf{v}(0) = \mathbf{v}_0 = 80\mathbf{j} + 80\mathbf{k}.$$

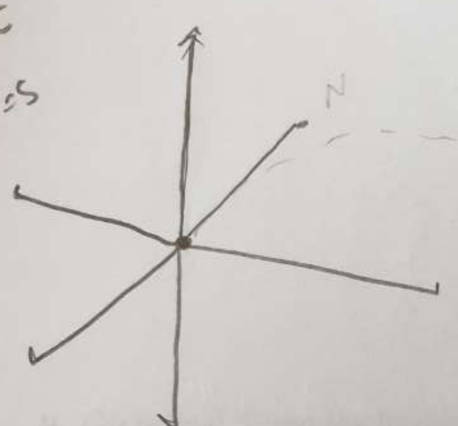
The spin on the ball causes an additional eastward acceleration of 2 ft/s^2, so that the acceleration is

$$\mathbf{a}(t) = 2\mathbf{i} - 32\mathbf{k}.$$

(a) When does the ball hit the ground, (b) at what location, and (c) with what speed? (Note three questions. Please circle your answers.)

$\frac{16}{3} = t$

$\frac{16}{3} = s$



N (+y)

$v(0) = v_0 = 80j + 80k$

$a(t) = 2i - 32k$

$x_0 = (0,0,0)$

$x_f = (x, y, z)$

a) $x = v_0 t + \frac{1}{2} a t^2$

$x = (80j + 80k)t + \frac{1}{2}(2i - 32k)(t)^2$

$2ax = (v_t) - (v_0)^2$

$= \int_0^{16/3} \sqrt{4t^2 + 80^2 + 32^2 t^2 + 80^2}$

$\int_0^{16/3} \sqrt{1020t^2 + 2(80)^2}$

$\int 2i - \int 32k$

$2t\hat{i} + 80j(-32 + 80)\hat{k} = v(t)$

$v(t) = 2t\hat{i} - 32t\hat{k} + C$

$v(0) = 80j + 80k = 2t\hat{i} - 32t\hat{k} + C$

$80j + 80k = C$

$2t\hat{i} + 80\hat{j} + (-32t + 80)k = v(t)$

$2t + 80 - 32t + 80 = 0$

$-30t + 160 = 0$

$\frac{160}{30} = 30t$

8. (10 points) What are the domain and range of the function $f(x, y) = \sin(\ln(x^2 - y^2 + 1))$?

Domain: $x^2 - y^2 + 1 > 0$ ln $\neq$ 0 or any negative #

Range: $-1 \leq z \leq 1$ sin fluctuates from -1 to 1

9. (10 points) Given the function $f(x, y) = \frac{1}{x^2+4y+1}$, find the partial derivatives f_x, f_y, f_{xx}, f_{xy}, and f_{yy}.

$$f(x,y) = \frac{1}{x^2+4y+1}$$

$$f_x = \frac{-(2x)}{(x^2+4y+1)^2} \qquad f_y = \left(\frac{-(4)}{(x^2+4y+1)^2}\right)$$

$$f_{xx} = \frac{-2(x^2+4y+1)^2 - (-2x)\,2(x^2+4y+1)(2x)}{((x^2+4y+1)^2)^2}$$

$$f_{yy} = \frac{-(-4)2(x^2+4y+1)(4)}{((x^2+4y+1)^2)^2}$$

$$f_{xy} = \frac{-(-2x)2(x^2+4y+1)(4)}{((x^2+4y+1)^2)^2}$$

These can be simplified