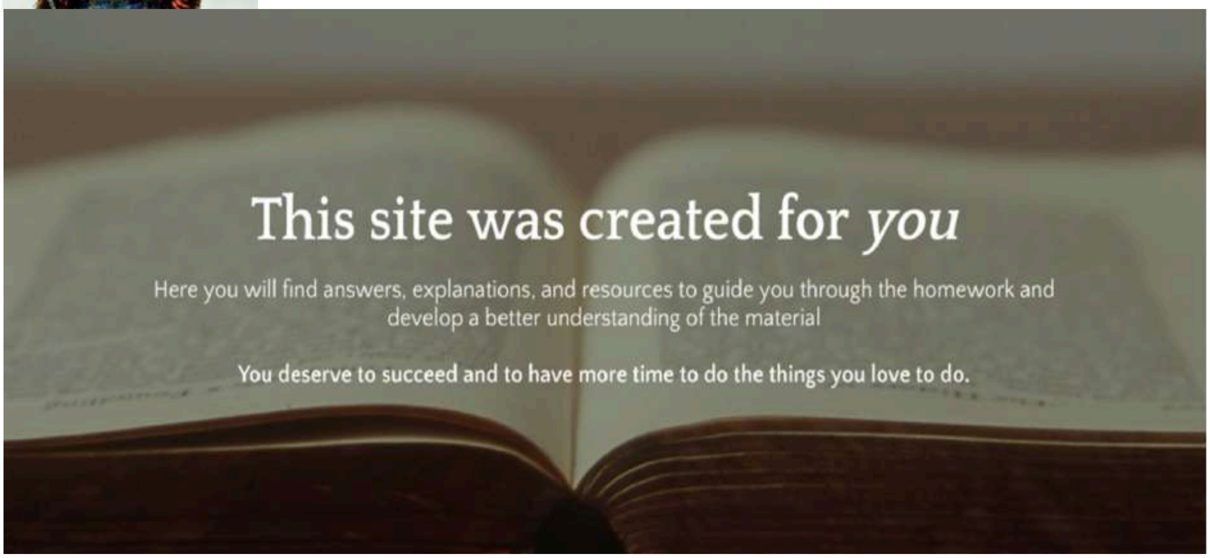
Highlander Help



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Good Luck!

Math 211: Calculus IIIA Common Exam 2 April 5, 2017

Name: (print) _

Student ID number: _

Section Number:

Signature*: _

*My signature affirms that this examination is completed in accordance with the NJIT Academic Integrity Code.

Instructions:

Please complete the problems on the following pages in the space provided. If you need additional space to work, please use the back of the previous page. All work must be shown in order to receive full credit. Answers without explanation will receive *no* credit. The use of books, notes, calculators, or any other external sources of information is not permitted during this examination.

Question	Points	Score	
1	15	10	-5
2	15	15	
3	15	15	
4	15	15	
5	15	12	-3
6	15	15	
7	10	10	
Total:	100	87	92

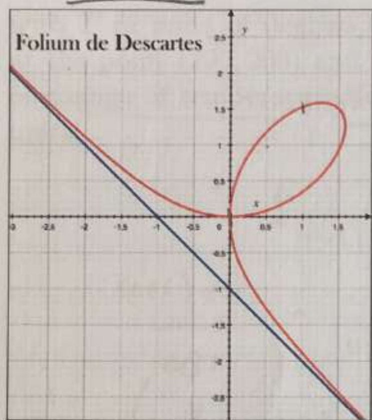
92

~~87~~

points dont matchup

1^3+1^3-3

1. (15 points) The folium of Descarte is the curve given by $x^3 + y^3 - 3xy = 0$, pictured here. Use the chain rule and implicit partial differentiation rule to determine the slope of this curve at any point (x, y) where the tangent is not vertical. At what points is the slope vertical?



slope vertical y no change in x

slope >

slope $= 0$

$F_y = 3y^2 - 3x$

$F_x = 3x^2 - 3y$

$w = x^3 + y^3 - 3xy$

$$\frac{dx}{dy} = \frac{-F_y}{-F_x} \qquad \frac{-(3y^2+3x)}{3x^2-3y} \qquad \boxed{\frac{-3y^2+3x}{3x^2-3y}}$$

10/15

slope of curve @ any pt (x,y)

$$\frac{dy}{dx} = \frac{-F_x}{F_y} \qquad \frac{-(3x^2-3y)}{3y^2-3x} \qquad \boxed{\frac{-3x^2+3y}{3y^2-3x}}$$

$1^3+1^3-3 \neq 0$

$$\frac{-3y^2+3x}{3x^2-3y} = 0 \qquad \frac{-3x^2+3y}{3y^2-3x} = 0$$

slope is vertical

$(1,1)$ DNE not part of original equation

$\boxed{(0,0)}$

$-3y^2+3x=0$

$3x = 3y^2$

$x = y^2$

$-3x^2+3y=0$

$x^2 = y$

$(y^2)^2 = y$

$y^4 = y$

$y=0$ $x=$

$y=1$ $x=$

2. (15 points) Suppose the temperature is given by

directional derivative

$$T(x,y) = \frac{1}{180}\left(7400 - 4x - 9y - (0.03)xy\right),$$

with T in units of degrees Celsius and distance in kilometers. If you are standing at the point $(200, 200)$ and walking northeast in the direction $\mathbf{v} = \langle 3, 4\rangle$, what rate of change of temperature do you experience with respect to position? Don't forget units!

$f_{(x,y)}\Big|_{P_0} \cdot u$

$\frac{-4}{180} - \frac{6}{180} \quad \frac{-1}{18}$

$d_x = \frac{1}{180}(-4 - .03y) \qquad \frac{1}{180}(-4 - .03(200))$

$d_y = \frac{1}{180}(-9 - .03x) \qquad \frac{1}{180}(-9 - .03(200))$

$\frac{1}{180}(-9-6)$

$\frac{-15}{180}$

$\frac{(3, 4)}{\sqrt{3^2+4^2}}$

$\langle -\frac{1}{18}\hat{\imath}, -\frac{15}{180} \rangle \cdot \langle \frac{3}{5}\hat{\imath}, \frac{4}{5}\hat{\jmath} \rangle$

$-\frac{1}{18}\left(\frac{3}{5}\right) + \frac{-3}{180}\left(\frac{4}{1}\right)$

$\frac{-3(2)}{(2)18(5)} + \frac{-12}{180}$

$\frac{-18}{180}$

$\boxed{-\frac{1}{10}}$ $\boxed{°C \text{ per Km}}$

18
5
90
90
$\overline{}$

3. (15 points) Find an equation for the tangent plane to the surface $z = 5 - 2x^2 - y^2$ through the point $(1, 1, 2)$.

$0 = 5 - 2x^2 - y^2 - z$

d_x $-4x$ $-4(1)$ -4

d_y $-2y$ $-2(1)$ -2

d_z -1 -1

$-4(x-1) - 2(y-1) - 1(z-2) = 0$

$-4x + 4 - 2y + 2 - z + 2 = 0$

$\boxed{-4x - 2y - z = -8}$

15

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d_x $-4x$ $-4(1)$ -4

d_y $-2y$ $-2(1)$ -2

d_z -1 -1 -1

$-4(x-1) - 2(y-1) - (z-2) = 0$

$-4x + 4 - 2y + 2 - z + 2 = 0$

$-4x - 2y - z = -8$

SP = saddle point

M = maximum

m = minimum

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4. (15 points) Show that the function $f(x, y) = x^2 + kxy + y^2$ has a critical point at $(0, 0)$ for all values of k. For what values of k is this a saddle point? For what values of k is it a local minimum? A local maximum? For what values is the second derivative test inconclusive?

① ② ③ ④ ⑤

$2x = -ky$

$\frac{2x}{-y} = k$

$|k| = 2$ ⑤ test is inconclusive cannot = 0

f_x: $2x + ky = 0$

$f_{xx} = 2$

Saddle point if $f_{xx}f_{yy} - f_{xy}^2 < 0$ ②

$|k| > 2$

f_y: $kx + 2y = 0$

$f_{yy} = 2$

$f_{xx}f_{yy} - f_{xy}^2 > 0$ $\boxed{\text{minimum}}$ if $f_{xx} > 0$ $|k| < 2$ ③

$f_{xx}f_{yy} - f_{xy}^2 > 0$ $\boxed{\text{maximum}}$ if $f_{xx} < 0$ never ④

$\left(\frac{2x}{-y}\right)x + 2y = 0$

$\frac{2x^2}{y} = 2y$

$2y^2 = 2x^2$

$y^2 = x^2$ ①

$(0,0)$ will always be a critical point no matter the k

$\pm 1 = k$

	x	y	f_{xx}	f_{yy}	f_{xy}/f_{yx} (continuous)	Δ	k
	0	0	2	2	k		
SP						$\Delta < 0$	$\|k\| > 2$
m						$\Delta > 0$	$\|k\| < 2$
M						$\neq$	$\|k\| \neq$
Inconclusive						$\Delta = 0$	$\|k\| = 2$

5. (15 points) Find the absolute maximum and minimum of the function

$$z = x^2 + y^2 - 2x - y$$

on the triangular region with corners at $(0, 0)$, $(2, 0)$, and $(0, 2)$.

$f(x) = 2x - 2 = 0$ $x = 1$

$f(y) = 2y - 1 = 0$ $y = \frac{1}{2}$

$f(z) = 1$

$(1, \frac{1}{2}) = -1.25$ absolute minimum

$(0, 0) = 0$

$(2, 0) = 0$

$(0, 2) = 2$ absolute Maximum

Boundary -3

12/15

$1^2 + (\frac{1}{2})^2 - 2(1) - (\frac{1}{2})$

$1 + \frac{1}{4} - 2 - \frac{2}{4}$

$1.25 - 2.5$

-1.25

6. (15 points) Find the maximum and minimum of the function $f(x, y) = xy$ on the circle $x^2 + y^2 = 2$. (Not the disk, the circle!)

$f = xy$

constraint $= x^2+y^2=2$

$f_x = y$

$f_y = x$

$g_x = 2x$

$g_y = 2y$

$\lambda 2x = y$

$\pm \frac{1}{2}(2)x = y$

$\boxed{x = \pm y}$

$x^2+y^2=2$

$y^2+y^2=2$

$2y^2=2$

$\boxed{y = \pm 1}$

$\lambda 2y = x$

$\lambda 2(\lambda 2x) = x$

$\lambda^2 4x = x$

$\sqrt{\lambda^2} = \sqrt{\frac{1}{4}}$

$\boxed{\lambda = \pm \frac{1}{2}}$

Maximum = 1

(1, 1)

(−1, −1)

minimum = −1

(−1, 1)

(1, −1)

7. (10 points) Calculate $\boxed{x = \pm 1}$

$$\iint_R y \cos x \, dA$$

where R is the rectangle satisfying $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$ and $0 \leq y \leq 1$.

$$\int_{-\pi/2}^{\pi/2} y \cos x \, dx$$

$$\Big[y \sin(x) \Big]_{-\frac{\pi}{2}}^{\pi/2}$$

$y \sin(\frac{\pi}{2}) - y \sin(-\frac{\pi}{2})$

$y + (+y)$

$$\int_0^1 2y \, dy$$

sin(x)

cos(x)

−sin(x)

−cos(x)

sin(x)

cos(x)

$\sin(\frac{\pi}{2})$

1

$\sin(-\frac{\pi}{2})$

−1

$\frac{2y^2}{2}$

$\Big[y^2 \Big]_0^1$

$\boxed{1}$