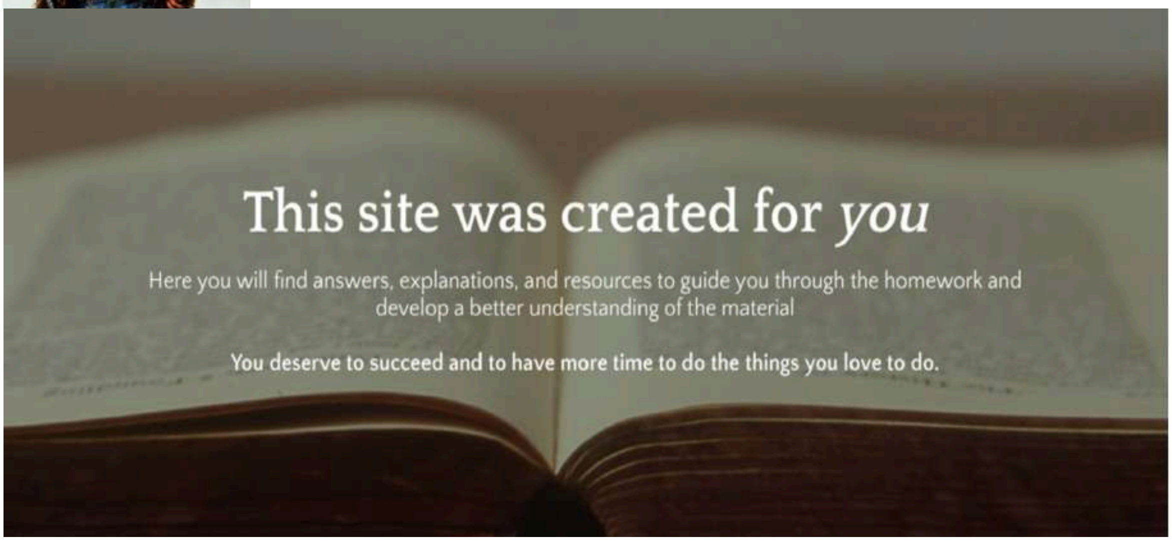
Highlander Help



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Good Luck!

13/20

$\overrightarrow{P_0P} \cdot n$

$A(x-$
$B(y-$
$C(z-$
$=0$

Quiz

MATH211-002, CALCULUS III, SPRING 2017

NAME:

Problem 1.(10 points.) Find the equation of the plane through $P_o(2,4,5)$ perpendicular to the line

$$x = 5 + t, \quad y = 1 + 3t, \quad z = 4t$$

.

$P(5,1,0)$

$n\ (1,3,4)$ ✓

~~$2(5+t) + 4(1+3t) + 5(4t) = 0$~~

~~$10 + 2t + 4 + 12t \quad 20t = 0$~~

~~$34t = -14/34$~~

~~$t = -7$~~

3/10

$(5-2) \quad (1-4) \quad (0-5)$

$(+3i, -3j, -5k) \cdot (1, 3, 4)$

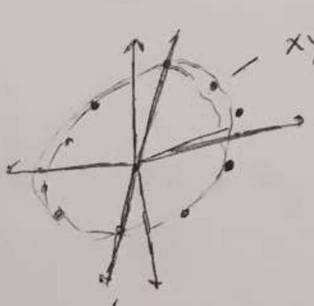
$\overrightarrow{P_0P} \cdot n$

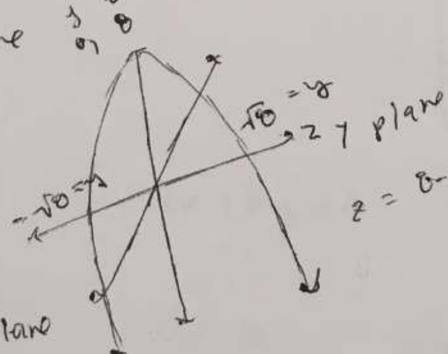
$3(1) \quad -3(3) \quad (-5)(4)$

$\boxed{3\hat{i} - 9\hat{j} - 20\hat{k}}$ ← eqn of a vector, not a plane.

Problem 2.(10 points.) For the following equation, describe the cross-sections with the three coordinate planes and sketch the quadratic surface (as best you can). Consider any intersections with coordinate axes when sketching.

$$z = 8 - x^2 - y^2$$





$z = 8 - y^2$

$0 = 8 - x^2 - y^2$

$8 - y^2 = x^2$

$8 - 2^2 = x^2$

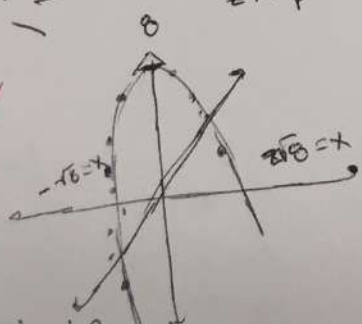
$8 - \sqrt{7}^2 = 1^2$

x y 0
2, 2

1, $\sqrt{7}$

shape =

✓



z x

$z = 8 - x^2$

$1 = 8 - 1^2$

x	z
1,	7
0,	8
2	4
3	-1

8 - 4
8 - 9

hyperboloid of one sheet?

✗ elliptical paraboloid

10/10

great work.

55

M211

Quiz1

1. Find the unit vector obtained by rotating the vector (0,1) by 120 deg. Counterclockwise.
2. Find the vector $5u - v$ if $u = (1,1,-1)$ and $v = (2,0,3)$.
3. Given a triangle defined by the three points $A = (-1,0)$, $B = (2,1)$ and $C = (1,-2)$ find the cosine of the angle between AB and AC
4. For extra credit:

Suppose that AB is the diameter of a circle with center O and that C is a point on one of the two arcs joining A and B. Show that CA and CB are orthogonal. (see the diagram)

$x^2+y^2=4$

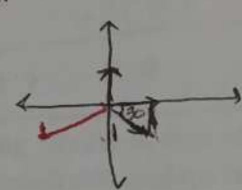
$1^2+y^2=4$

dot product = 0 perpendicular

1)

-10

$\boxed{\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)}$



$(1,\sqrt{3})$

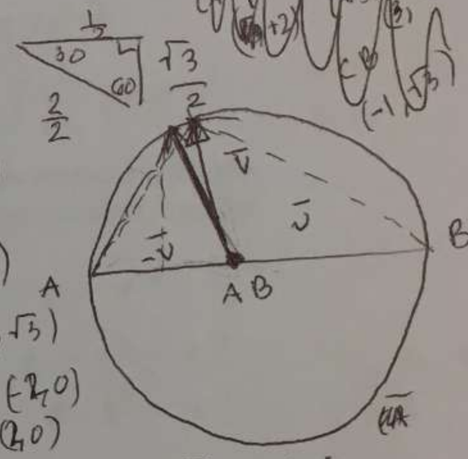
$(-1,\sqrt{3})$

4)

$C=(1,\sqrt{3})$

$A=(-2,0)$

$B=(2,0)$



$\overline{CA} \cdot \overline{CB} = 0$

$\theta = \cos^{-1}\left(\frac{-1+\sqrt{3}}{\sqrt{4}\ \sqrt{4}}\right)$

$\theta = \cos^{-1}\left(\frac{\sqrt{3}-1}{4}\right)$

dot product

2) $5(1,1,-1)$ $-(2,0,3)$

$(5,5,-5)$ $(-2,0,-3)$

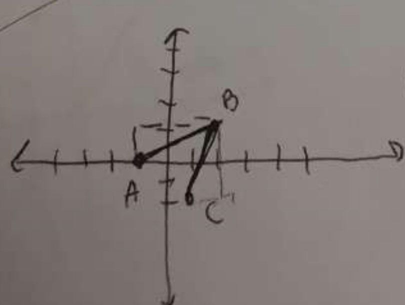
$(5-2)\hat{i} + (0+5)\hat{j} + (-5-3)\hat{k}$

$\boxed{\langle 3i + 5j - 8k \rangle}$

3)

$\overline{AB} \cdot \overline{AC}$ X

$(3,1)$ $(1,3)$



$\cos^{-1}\left(\frac{3(1)+3(1)}{\sqrt{10}\ \sqrt{10}}\right)$

3^2+1

$\theta = \cos^{-1}\left(\frac{6}{10}\right)$

$\boxed{\theta = \cos^{-1}\left(\frac{3}{5}\right)}$

/ o v

Quiz 2

1. Find dw/dt at t = 0 if $w = \sin(xy + \pi)$, $x = e^t$, and y = ln(t+1)
2. For extra credit:

Show that if w = f(s) is any differentiable function of x and if s = y + 5x, then

$$\frac{\partial w}{\partial x} - 5\frac{\partial w}{\partial y} = 0$$

$\ln(1) = 0$

$w = \sin(xy + \pi)$

1) $w = \sin(e^t(\ln(t+1)) + \pi)$

$\frac{dw}{dt} = \cos(e^t(\ln(t+1)) + \pi)\left(e^t(\ln(t+1)) + e^t\left(\frac{1}{t+1}\right)\right)$

$t = 0$ $\cos(e^0\cancel{(\ln(0+1))} + \pi)\ (e^0\cancel{(\ln 1)} + e^0(\frac{1}{1})$

$\cos(\pi)$ (1)

$-1(1)$

$\boxed{-1}$

10. ✓

$S = y + 5x$

2) $\frac{dw}{dx} = 5$ $\frac{dy}{dx} = 1$

$5 - 5(1) = 0$

$0 = 0$

* 1(5

Quiz 4

?
1. What is the largest value of f(x,y,z) = xyz that the directional derivative can take at the point (1,1,1)?
2. For the surface: $z = \ln(x^2+y^2)$, find the equation of the tangent plane at the point (0,1,0)

For extra credit:

3. Find a linear approximation to the function w = f(x,y,z) = (sin xy)/z at the point (2,0,1) Apply the linearization to approximating f(2.1, -0.1,1.1)

$f(x,y,z) = xyz$

$|\nabla f|$

1) $f_x = yz = f_x(1,1,1) = 1$

$f_y = xz = f_y(1,1,1) = 1 =$ $\left| \frac{1}{\sqrt{3}}\hat{\imath}, \frac{1}{\sqrt{3}}\hat{\jmath} + \frac{1}{\sqrt{3}}\hat{k} \right|$

$f_z = xy = f_z(1,1,1) = 1$

2) $0 = \ln(x^2+y^2) - z$

$f_x = \frac{1}{x^2+y^2}(2x) = f_{x\,0,1,0} = 0$

$f_y = \frac{1}{x^2+y^2}(2y) = f_{y\,0,1,0} = 2$

$f_z = -1 = f_{z\,0,1,0} = -1$

$0(x-0) + 2(y-1) - 1(z-0) = 0$

$2y - 2 - z = 0$

$\boxed{2y - z = 2}$

3) $f(x) = \frac{\sin(xy)}{z}$ $f(x)_{2,0,1}$ $\frac{\sin(2\cdot 0)}{(1)} = 0$

$f_x = \frac{1}{z}\, y\cos(xy) = f_{x\,2,0,1} = 0$ $0 + 0(.1) + 2(-.1) + 0(.1)$

$f_y = \frac{1}{z}\, x\cos(xy) = f_{y\,2,0,1} = 2$

$f_z = \frac{-\sin xy}{z^2} = f_{z\,2,0,1} = 0$ $\boxed{-.2}$

Quiz 5

75

1. $f(x,y) = x^2 + 2xy$

Find the stationery point(s) and classify them

2. Use Lagrange multipliers to find the absolute maxima and minima of:

$f(x,y) = xy$

subject to the constraint:

$x^2 + y^2 = 10$

$x^2 = 10 - y^2$

$x = \pm\sqrt{10-y^2}$

$\nabla f = \lambda \nabla g$

$y\hat{i} + x\hat{j} = \lambda(2x\hat{i} + 2y\hat{j})$

$x^2+y^2=10$

$y = 2x\lambda \quad x = 2y\lambda$

$\frac{y}{2} = x\lambda \quad x = 2y\lambda$

$\frac{y}{2} = (2y\lambda)\lambda$

$\frac{1}{2} = 2\lambda^2$

$\frac{1}{4} = \lambda^2$

$\pm\left(\frac{1}{4}\right)^{\frac{1}{2}} = \lambda$

$\frac{y}{2} = (10-y^2)^{\frac{1}{2}}$

1) $f_x = 2x + 2y$ $\quad 2x + 2y = 0 \quad y = 0$

$f_y = 2x$ $\quad 2x = 0 \quad x = 0$

f_{xx} 2

f_{yy} 0

f_{xy} 2

	x	y	f_{xx}	$f_{xx}f_{yy} - f_{xy}^2$	
(0,0)	0	0	2	$-(2)^2$	S.P

50

2. $f(x,y) = xy$ constraint $x^2+y^2=10$

2f

$f_x \quad y = 0$ $\quad y\,x = \lambda(2x\,2y)$

$f_y \quad x = 0$

$f(0,y)$ $\quad f(x,0)$ $\quad y = \lambda 2x$ $\quad x = \lambda 2y$

$x^2+y^2=10$ $\quad y^2=10$ $\quad \frac{x}{2y} = \lambda$

$x^2=10$ $\quad (0, \pm\sqrt{10})$ $\quad \frac{2x}{y} =$

$(0, \pm\sqrt{10})$

$x^2+y^2=10$

$4xy = xy$

$x^2 = 10-y^2$

$4(-\sqrt{10-y^2})y = (-\sqrt{10-y^2})y$

$x = \pm\sqrt{10-y^2}$

$x=0$

$y=0$ $\quad y=0$ $\quad x=0$ $\quad y = \pm\sqrt{10-y^2}$

$x=\sqrt{10}$ $\quad x=-\sqrt{10}$ $\quad y=+\sqrt{10}$ $\quad y=-\sqrt{10}$

0 $\quad$ 0

AM 25

$f(x,y)$ $\quad$ 0 $\quad$ 0

$x \quad \pm\frac{\sqrt{10}}{2}$

$y \quad \pm\frac{\sqrt{10}}{2}$

Am -25

3 $\quad f(x,y) = x^2+2y-x$ $\quad x^2+y^2 \leq 4$

$2x-1=0$ $\quad \frac{1}{2} = x$

$2x$

$f_x \quad 2x-1$

$y=0$ $\quad 2y$ $\quad 0 \;\frac{1}{2}$

$f_y \quad 2y$

$\left(\frac{1}{2}, 0\right)$ $\quad -\frac{1}{4}$ m

$2x-1 = 2x\lambda$ $\quad 2y = 2y\lambda$

$1 = \lambda$

$\frac{2x-1}{2x} = \lambda$

$\frac{2x-1}{2x} = 1$

Quiz 7

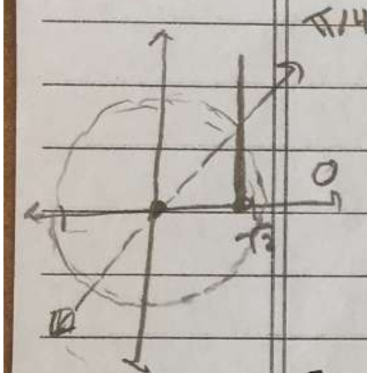
Do either problem. You may do the other for extra ~~cr~~ credit.

1. For the following integral

Sketch the region of integration,

Convert the integral to polar

coordinates and integrate.



10

$$\int_0^1 \int_x^{\sqrt{2-x^2}} x \, dy \, dx$$

$\boxed{\dfrac{\sqrt{2}}{6}}$

$y^2 = 2 - x^2$

$$\int_0^{\pi/4} \int_0^{\sqrt{2}} r^2 \cos\theta \, dr \, d\theta$$

2. Find the volume of the region bounded ~~by~~ in the first octant

by the coordinate planes,

the plane $y + z = 2$ and the cylinder $x = 4 - y^2$

$$\int_0^2 \int_0^{4-y^2} \int_0^{2-y} dz \, dx \, dy$$

$$\int_0^{4-y^2} 2 - y \, dx$$

$$\int_0^2 2x - yx$$

$$\int_0^2 2(4-y^2) - y(4-y^2) \, dy$$

$$\left[8y - \frac{2y^3}{3} - \frac{4y^2}{2} \right]_0^2$$

$8 - 2y^2 - 4y + y^3$

$8y - \dfrac{2y^3}{3} - \dfrac{4y^2}{2} + \dfrac{y^4}{4}$

$16 - \dfrac{2(8)}{3} - 8 + 4$

$\dfrac{8(3)\,12}{(3)} - \dfrac{16}{3}$

$\boxed{\dfrac{20}{3}}$